LIBERTY PAPER SET

STD. 10 : Mathematics (Basic) [N-018(E)]

Full Solution

Time : 3 Hours **ASSIGNTMENT PAPER 3**

Section-A

1. (C) 3 **2.** (C) $b^2 - 4ab > 0$ **3.** (C) –77 **4.** (B) 8 **5.** (B) 50° **6.** (A) 6 **7.** $a^3\ b^2$ **8.** 2 **9.** 1 **10.** –1 **11.** 1 **12.** 3 **13.** True **14.** False **15.** False **16.** False **17.** –15 **18.** 8 **19.** $\frac{2}{5}$ **20.** 25 **21.** (b) πrl **22.** (c) $\frac{4}{3}\pi r^3$ **23.** (c) πr^2 **24.** (a) $\frac{\pi r\theta}{180}$

Section-B

25. Let, $P(x) = 0$

$\therefore\ x^2 + 2x - 8 = 0$

$\therefore\ x^2 + 4x - 2x - 8 = 0$

$\therefore\ x(x + 4) - 2(x + 4) = 0$

$\therefore\ (x + 4)(x - 2) = 0$

$\therefore\ x + 4 = 0$ OR $x - 2 = 0$

$\therefore\ x = -4$ OR $x = 2$

26. Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$\therefore\ \alpha + \beta = \frac{1}{4} = \frac{-b}{a}$ and $\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$

$\therefore\ a = 4,\ b = -1$ and $c = -4$

So, one quadratic polynomial which fits the given conditions is $4x^2 - x - 4$. You can check that any other quadratic polynomial that fits these conditions will be of the form $k(4x^2 - x - 4)$, where k is real.

27. $2x^2 - 6x + 3 = 0$

$\therefore\ a = 2,\ b = -6$ and $c = 3$

$\therefore\ b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12$

Here $b^2 - 4ac > 0$, therefore, there are distinct real roots exist for given equation.

Now, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore\ x = \frac{-(-6) \pm \sqrt{12}}{2 \times 2}$

$\therefore\ x = \frac{6 \pm 2\sqrt{3}}{4}$

$\therefore\ x = \frac{3 \pm \sqrt{3}}{2}$

Therefore, roots of given equation : $\frac{3+\sqrt{3}}{2},\ \frac{3-\sqrt{3}}{2}$

28. Numbers divisible by 3 are,

102, 105, 108,... 999.

So, here a = 102, d = 105 – 102 = 3, a*n* = 999

we have, a*n* = a + (n – 1)d

$\therefore$ 999 = 102 + (n – 1) (3)

$\therefore$ 999 – 102 = (n – 1) (3)

$\therefore$ 897 = (n – 1)(3)

$\therefore$ $\frac{897}{3} = n - 1$

$\therefore$ 299 = n – 1

$\therefore$ n = 299 + 1

$\therefore$ n = 300

So, there are total 300 numbers of three digits which are divisible by 3.

29. $a = 7$, $d = 13 - 7 = 6$, $a_n = 205$, n = ______

$a_n = a + (n - 1)\, d$

$\therefore 205 = 7 + (n - 1)\, 6$

$\therefore 205 - 7 = (n - 1)\, 6$

$\therefore \frac{198}{6} = n - 1$

$\therefore n - 1 = 33$

$\therefore n = 34$

Hence, this given series has 34 terms in it.

30. The distance between P (2, –3) and Q (10, y) is 10 units.

$\therefore$ PQ = 10

$\therefore$ $PQ^2 = (10)^2$

$\therefore$ $(2 - 10)^2 + (-3 - y)^2 = 100$

$\therefore$ $64 + 9 + 6y + y^2 - 100 = 0$

$\therefore$ $y^2 + 6y - 27 = 0$

$\therefore$ $y^2 + 9y - 3y - 27 = 0$

$\therefore$ $y(y + 9) - 3(y + 9) = 0$

$\therefore$ $(y + 9)\,(y - 3) = 0$

$\therefore$ $y + 9 = 0$ OR $y - 3 = 0$

$\therefore$ $y = -9$ OR $y = 3$

Hence, $y = -9$ and 3.

31. Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5)

$\therefore$ AP = BP

$\therefore$ $AP^2 = BP^2$

$\therefore (x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

$\therefore x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$

$\therefore -14x - 2y + 50 = -6x - 10y + 34$

$\therefore -14x + 6x - 2y + 10y = 34 - 50$

$\therefore -8x + 8y = -16$

$\therefore x - y = 2$

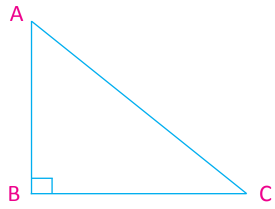
Therefore, $x - y = 2$ is the required relation.

32. $= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4}$

$= 2$

33.



Δ ABC is a right angle triangle, $\angle B = 90°$

$15\ cot\ A = 8$

$\therefore\ cot\ A = \frac{8}{15}$

$\therefore\ \frac{AB}{BC} = \frac{8}{15}$

$\therefore\ \frac{AB}{8} = \frac{BC}{15} = k$, where k is a positive real number

$\therefore$ AB = $8k$, BC = $15k$

According to Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$\therefore\ AC^2 = (8k)^2 + (15k)^2$

$\therefore\ AC^2 = 64k^2 + 225k^2$

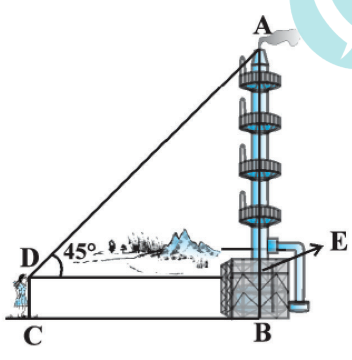
$\therefore\ AC^2 = 289k^2$

$\therefore\ AC = 17k$

$\therefore\ sin\ A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$ and

$sec\ A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

34.



Here, AB is the chimney, CD the observer $\angle$ADE the angle of elevation.

In Δ ADE, $\angle$E right angled

AB = AE + BE = AE + 1.5 and DE = CB = 28.5 m

Now, $tan\ 45° = \frac{AE}{DE}$

$\therefore\ 1 = \frac{AE}{28.5}$

$\therefore$ AE = 28.5

So, the height of chimney = AB = AE + 1.5 = 28.5 + 1.5 = 30 m.

35. Total surface area of a solid

= CSA of a cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$

36. Total surface area of a cube

$= 6l^2$

$= 6 \times (5)^2$

$= 6 \times 25$

$= 150$ cm^2

37. mean $(\bar{x}) = a + \dfrac{\sum fidi}{\sum fi}$

$= 30 + \dfrac{-26}{13}$

$= 30 - 2$

$= 28$

38. $2x + 3y = 46$ ...(1)

$3x + 5y = 74$ ...(2)

Multiply eq. (1) by 3 & eq^n (2) by 2 and subtract them,

$6x + 9y = 138$

$6x + 10y = 148$

$-\quad -\qquad -$

$\therefore\ -y = -10$

$\therefore\ y = 10$

Put $y = 10$ in (1),

$2x + 3y = 46$

$\therefore\ 2x + 3(10) = 46$

$\therefore\ 2x + 30 = 46$

$\therefore\ 2x = 46 - 30$

$\therefore\ 2x = 16$

$\therefore\ x = \dfrac{16}{2}$

$\therefore\ x = 8$

$x = 8,\ y = 10$

39. In two positive integers, biggest, positive integer is x.

As per condition,

biggest integer – smallest integer = 5

$\therefore$ x – smallest integer = 5

$\therefore$ smalletst integer = $x - 5$

As per condition,

$$\frac{1}{x-5} - \frac{1}{x} = \frac{1}{10}$$

$\therefore$ $10x - 10\ (x - 5) = x\ (x - 5)$

$\therefore$ $10x - 10\ x + 50 = x^2 - 5x$

$\therefore$ $x^2 - 5x - 50 = 0$

$\therefore$ $(x - 10)\ (x + 5) = 0$

$\therefore$ $x - 10 = 0$ OR $x + 5 = 0$

$\therefore$ $x = 10 = 0$ OR $x = -5$

But $x = -5$ is not possible

$\therefore$ $x = 10$

$\therefore$ Biggest positive integer $= x = 10$ and

Smallest positive integer $= x - 5 = 10 - 5 = 5$

40. The positive integers that are divisible by 6 are 6, 12, 18, 24, ...

$a = 6,\ d = 6,\ n = 40$

$$S_n = \frac{n}{2}\ [2a + (n - 1)d]$$

$$\therefore S_{40} = \frac{40}{2}\ [2(6) + (40 - 1)\ 6]$$

$$= 20\ (12 + 234)$$

$$= 20\ (246)$$

$$\therefore S_{40} = 4920$$

41. $$\left(\frac{a}{3},\ 4\right) = \left(\frac{-6+(-2)}{2},\ \frac{5+3}{2}\right)$$

$$\therefore \left(\frac{a}{3},\ 4\right) = \left(\frac{-8}{2},\ \frac{8}{2}\right)$$

$$\therefore \left(\frac{a}{3},\ 4\right) = (-4,\ 4)$$

$$\therefore \frac{a}{3} = -4$$

$\therefore$ $a = -4 \times 3 = -12$

42. Let (–4, 6) divide the line segment joining the points A (–6, 10) and B (3, –8) internally in the ratio $m_1 : m_2$ using the section formula, we get,

$$(-4,\ 6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2},\ \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)$$

$$\therefore -4 = \frac{3m_1 - 6m_2}{m_1 + m_2}$$

$\therefore$ $-4m_1 - 4m_2 = 3m_1 - 6m_2$

$\therefore$ $-4m_1 - 3m_1 = -6m_2 + 4m_2$

$\therefore$ $-7m_1 = -2m_2$

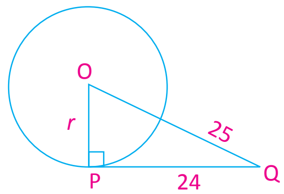
$\therefore$ $7m_1 = 2m_2$

$$\therefore \frac{m_1}{m_2} = \frac{2}{7}$$

$\therefore$ $m_1 : m_2 = 2 : 7$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7.

43.



Here, the tangent drawn to the circle from point Q. outside the circle with center O is PQ and Hence tangent point P and radious r.

In Δ OPQ, ∠P = 90°

∴ $OP^2 + PQ^2 = OQ^2$ (Pythagoras theorem)

∴ $r^2 + (24)^2 = (25)^2$

∴ $r^2 + 576 = 625$

∴ $r^2 = 625 - 576$

∴ $r^2 = 49$

∴ $r = 7$

Therefore, The radius of the circle is 7 cm.

44. Here, POQT is a quadrilateral, in which the opposite angles are complementry angles.

∴ ∠POQ + ∠PTQ = 180°

∴ 110° + ∠PTQ = 180°

∴ ∠PTQ = 70°

45. Here we get the information as shown in the table below using a = 225 and h = 50 to use the deviation method.

Daily expenditure (in ₹)	(f_i)	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100 – 150	4	125	– 2	– 8
150 – 200	5	175	– 1	– 5
200 – 250	12	225 = a	0	0
250 – 300	2	275	1	2
300 – 350	2	325	2	4
Total	$\Sigma f_i = 25$	–	–	$\Sigma f_i u_i = -7$

Mean $\bar{x}$ $= a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

∴ $\bar{x} = 225 + \frac{-7}{25} \times 50$

∴ $\bar{x} = 225 - 14$

$\bar{x} = 211$

So, mean daliy expenditure on food is ₹ 211.

46. Here, total number of cards = 52

(i) Suppose event A is the king of red colour card.

$$\therefore P(A) = \frac{\text{Number of king of red colour}}{\text{Total number of cards}}$$

$$\therefore P(A) = \frac{2}{52} = \frac{1}{26}$$

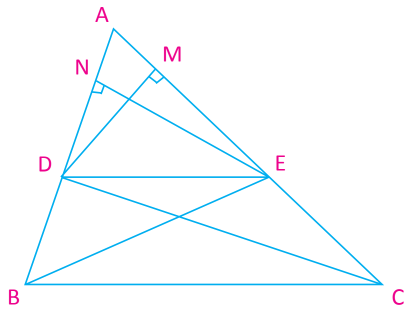
(ii) Suppose event B is no black spade card

$$\therefore P(B) = \frac{39}{52} = \frac{3}{4}$$

(iii) Suppose event C is queen of a heart card

$$\therefore P(C) = \frac{1}{52}$$

47.



Given: In ABC, a line parallel to side BC intersects AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Proof : Join BE and CD and also draw DM $\perp$ AC and EN $\perp$ AB.

Then, $ADE = \frac{1}{2} \times AD \times EN,$

$BDE = \frac{1}{2} \times DB \times EN,$

$ADE = \frac{1}{2} \times AE \times DM$ and

$DEC = \frac{1}{2} \times EC \times DM.$

$$\therefore \frac{ADE}{BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad ...(1)$$

$$\text{and } \frac{ADE}{DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad ...(2)$$

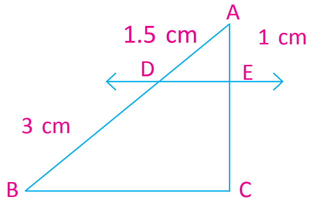
Now, Δ BDE and Δ DEC are triangles on the same base DE and between the parallel BC and DE.

then, BDE = DEC ...(3)

Hence from eq^n. (1), (2) and (3),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

48. (i)

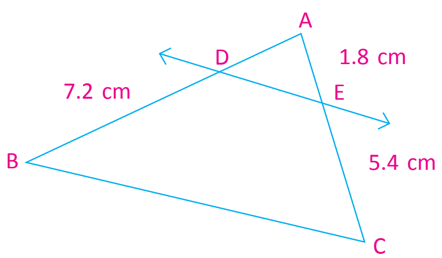


$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \text{(Theorem : 6.1)}$$

$$\therefore \frac{1.5}{3} = \frac{1}{EC}$$

$\therefore$ EC = 2 cm

(ii)



$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \text{(Theorem : 6.1)}$$

$$\therefore \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\therefore AD = \frac{1.8 \times 7.2}{5.4}$$

$\therefore$ AD = 2.4 cm

49. Given distance 360 km.

$\therefore$ Let the speed of train is x km/hr.

$\therefore$ Time for this speed = $\frac{360}{x}$ h

$\therefore$ Speed when increased by 5 km/h = $x + 5$

$\therefore$ Time for this speed = $\frac{360}{x+5}$ h

$$\therefore \frac{360}{x} - \frac{360}{(x+5)} = 1$$

$\therefore$ 360 $(x + 5)$ – 360 x = x $(x + 5)$

$$\frac{(360x + 1800 - 360x)}{x(x+5)} = 1$$

$$\frac{1800}{x(x+5)} = 1$$

$1800 = x(x + 5)$

$x^2 + 5x - 1800 = 0$

$x^2 + 45x - 40x - 1800 = 0$

$x(x + 45) - 40(x + 45) = 0$

$(x - 40)(x + 45) = 0$

$\therefore$ $x - 40 = 0$ OR $x + 45 = 0$

$\therefore$ $x = 40$ OR $x = -45$

But $x = -45$ is impossible

$\therefore$ $x = 40$

The speed of the train is 40 km/hr.

50. $7 - a = b - 7 = 23 - b = c - 23$

$\therefore\ b - 7 = 23 - b$

$\therefore\ b + b = 23 + 7$

$\therefore\ 2b = 30$

$\therefore\ b = 15$

Now, $7 - a = b - 7$

$\therefore\ 7 - a = 15 - 7$

$\therefore\ 7 - a = 8$

$\therefore\ a = 7 - 8$

$\therefore\ a = -1$

and $23 - b = c - 23$

$\therefore\ 23 - 15 = c - 23$

$\therefore\ 8 = c - 23$

$\therefore\ c = 23 + 8$

$\therefore\ c = 31$

51. Here the maximum class frequency is 8, and the class corresponding to this frequency is 3 – 5. So, the modal class is 3 – 5.

$\therefore\ l$ = Lower limit of modal class = 3

h = Class size = 2

f_1 = frequency of the modal class = 8

f_0 = frequency of class preceding the modal class = 7

f_2 = frequency of class succeeding the modal class = 2

Mode $\quad Z = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$\therefore\ Z = 3 + \left(\frac{8-7}{2(8)-7-2}\right) \times 2$$

$$\therefore\ Z = 3 + \frac{1}{7} \times 2 = 3 + \frac{2}{7} = 3 + 0.286$$

$\therefore\ Z = 3.286$

Therefore, the mode of the data above is 3.286.

52.

class	frequency (f_i)	cf
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$

Here, M = 28.5

$n = 60$

Median class = 20 – 30

l = lower limit of median class = 20

n = total frequency = 60

cf = cumulative frequency of class preceding the median class = $5 + x$

f = frequency of median class = 20

h = class size = 10

$$M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore\ 28.5 = 20 + \left(\frac{\frac{60}{2} - (5+x)}{20}\right) \times 10$$

$$\therefore\ 28.5 - 20 = \frac{(30 - 5 - x) \times 10}{20}$$

$$\therefore\ \frac{8.5 \times 20}{10} = 25 - x$$

$\therefore$ 17 = 25 – x

x = 25 – 17

x = 8

Now, $\sum f_i = n = 60$

$\therefore$ 45 + x + y = 60

$\therefore$ 45 + 8 + y = 60

$\therefore$ 53 + y = 60

$\therefore$ y = 60 – 53

$\therefore$ y = 7

Thus, x = 8 and y = 7.

53. Possible outcomes in throwing a dice are = 6 (1, 2, 3, 4, 5, 6)

Number of prime numbers = 3 (2, 3, 5)

Number lying between 2 and 6 = 3 (3, 4, 5)

Number of odd numbers = 3 (1, 3, 5)

(i) Suppose event A getting a prime number on the dice.

$$\therefore\ P(A) = \frac{\text{Number of prime number}}{\text{Total number of possible outcomes}}$$

$$\therefore\ P(A) = \frac{3}{6}$$

$$\therefore\ P(A) = \frac{1}{2}$$

(ii) Suppose event B getting a number lying between 2 and 6 on dice.

$$\therefore\ P(B) = \frac{\text{Number lying between 2 and 6}}{\text{Total number of possible outcomes}}$$

$$\therefore\ P(B) = \frac{3}{6}$$

$$\therefore\ P(B) = \frac{1}{2}$$

(iii) Suppose event C getting a number of odd number on dice.

$$\therefore\ P(C) = \frac{\text{Number of odd number}}{\text{Total number of possible outcomes}}$$

$$\therefore\ P(C) = \frac{3}{6}$$

$$\therefore\ P(C) = \frac{1}{2}$$

(iv) Suppose event D getting a whole numbers on the dice.

$$\therefore P(D) = \frac{\text{Number of a whole number}}{\text{Total number of possible outcomes}}$$

$$\therefore P(D) = \frac{6}{6}$$

$$\therefore P(D) = 1$$

54. Total number of possible outcomes = 8

(i) Suppose event A points to arrow 5.

$$\therefore P(A) = \frac{\text{Number of result that have 5}}{\text{Total number of outcomes}}$$

$$\therefore P(A) = \frac{1}{8}$$

(ii) Suppose event B is getting a prime number on the arrow. (2, 3, 5, 7)

$$\therefore P(B) = \frac{4}{8} = \frac{1}{2}$$

(iii) Suppose event C is getting a number greater than 4 is 4 (5, 6, 7, 8)

$$\therefore P(C) = \frac{4}{8} = \frac{1}{2}$$

(iv) Suppose event D is getting a number less than 3 is 2 (1, 2)

$$\therefore P(D) = \frac{2}{8} = \frac{1}{4}$$